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A Commentary by Harley Bassman:

The Convexity Maven

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Value Concepts from the Credit Suisse Trading Desk
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"The FVA: Forward Volatility Agreement"

Explained in full replicative detail **by Danielle N. Mohney**

Summary:

A Forward Volatility Agreement (FVA) is a forward on a vanilla swaption straddle. The buyer agrees to purchase a straddle on a specified date (the strike date) for a price that is determined today. The option becomes a standard option on strike date and is struck at the usual ISDA fixing time at the then current ATM forward rate.

Keeping the option "unstruck" until strike date gives the buyer concentrated and relatively constant exposure to Implied Volatility while mitigating exposure to the usually tricky delta, gamma, and theta inherent in a standard option. Thus, the FVA demands reduced management for a user wishing to hedge, or take a view on, changes in implied volatility. The FVA uniquely offers the risk management tool of *"Volatility as an Asset Class"*.

A Case Study Example: An investor believes that implied volatility looks cheap but does not have a strong view on the near-term actual volatility of interest rates. Moreover, he does not care to manage the "greek risk" inherent in owning options. Via an FVA, the investor can buy a six-month forward option on 1y10y (aka 6m1y10y) at XYZbps.

Today (T_0): Customer commits to buy a 1y10y straddle in 6m for a price of XYZbps.

Strike date ($T_{0.5}$): At the standard ISDA fixing time, the straddle is struck at the ATM forward rate. The Customer then has a few choices. He can be delivered a vanilla 1y10y straddle by paying the XYZbps premium as a "regular way" purchase. He can then hold the position or he can sell it at any time to Credit Suisse (or to another counter-party via an assignment). In the alternative, he can "cash settle" the contract with Credit Suisse versus the bid-side of the market. If this bid-side price is above his contracted price, the Customer is paid the difference, if it is lower, the Customer pays Credit Suisse. In this latter format, the FVA is a reduced management financial tool that can be employed to buy pure *"Volatility as an Asset Class".*

Expiry date ($T_{1.5}$): If the Customer elects to take delivery and hold the option until the final expiry date, he will exercise the payer or receiver side of the straddle into a 10y swap on this date. The swap will settle "regular way", t+2 from the expiry date.

Maturity date ($T_{11.5}$): The swap matures.

A Challenge: The FVA Replication Process

As a forward rate can be (mostly) replicated with spot rates, an FVA can be (partially) replicated using vanilla options. As such, it would be helpful to review the replication of forward rates.

The 1y10y rate, for example, can be replicated with 1y and 11y rates. One can gain a short exposure to the forward rate by paying the 11y rate and receiving the 1y rate, DV01-weighted using the following property:

$$DV01_{1,10} * r_{1,10} = DV01_{11} * r_{11} - DV01_1 * r_1$$

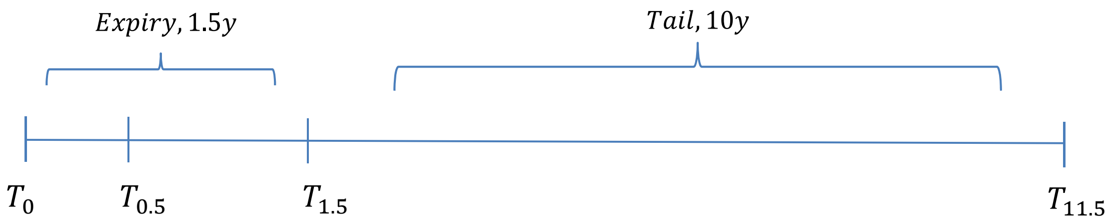
Thus, one could replicate the exposure of paying 100k/bp of the 1y10y rate by doing the following trades:

- Pay $\mathrm{DV01}_{11}/\mathrm{DV01}_{1,10}$ * 100k/bp of the 11y rate
- Rec $\mathrm{DV01}_{1}/\mathrm{DV01}_{1,10}$ * 100k/bp of the 1y rate

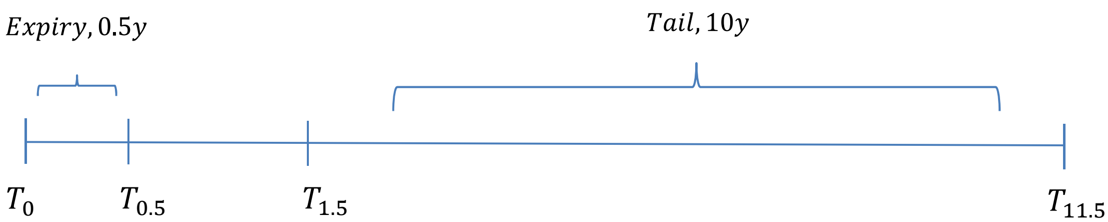
To replicate the FVA, we need to buy the variance from T_0 to $T_{1.5}$ and sell back the variance from T_0 to $T_{0.5}$ (we are locking in the expected diffusion of the rate from $T_{0.5}$, when the option is struck, to $T_{1.5}$, when it expires).

To do this, we buy a 1.5y10y straddle and sell a 0.5y expiry option on the same rate. This second option is an option on a forward rate – a 0.5y option on the 1y10y rate. An option on a forward rate is known as an (almost) vanilla "mid-curve" option.

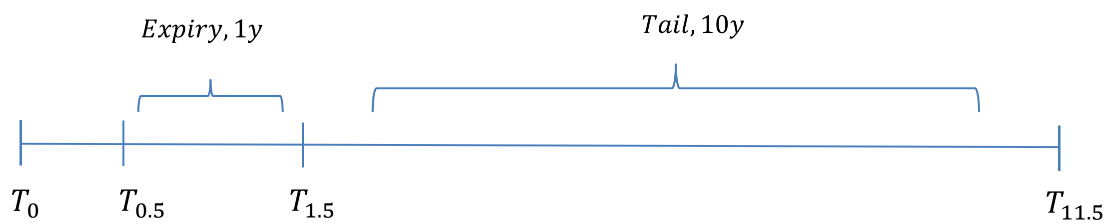
A graphical explanation might be helpful. We buy 1.5y into10y:



And sell 0.5y into the 1y10y forward rate:



To gain exposure to 1y10y swaption vol, 6m forward:



As such, one can gain exposure to 100k/nv of 6m forward 1y10y by doing the following:

-Buy 160k/nv 18m10y
-Sell 60k/nv 6m expiry options on the 1y10y rate (6m1y10y mid-curve)

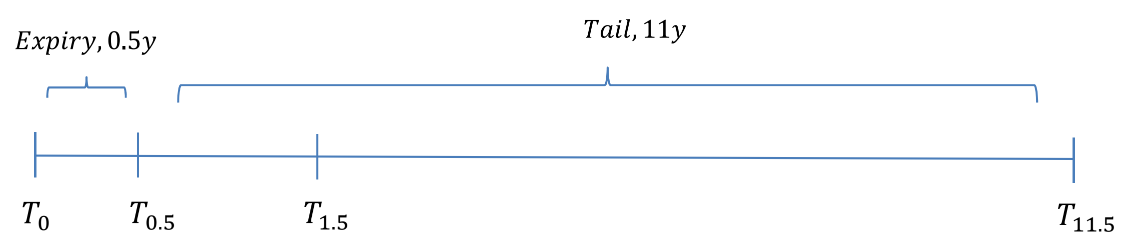
There is slippage here due to stochastic volatility and skew effects since these options are struck at a fixed-strike determined at T_0, while the FVA has no fixed-strike before the Strike Date. This is the most complete replication we can achieve without more exotic options. (The calculations for the ratios above are discussed in the Appendix of this primer.) **NOTE: The critical "value add" for the FVA product is that it can never be "fully replicated" via a limited series of fixed-strike options.**

We have now reduced the trade to two struck options; a vanilla option and an option on a forward rate (mid-curve option).

We can (approximately) replicate the mid-curve with vanilla options as well.

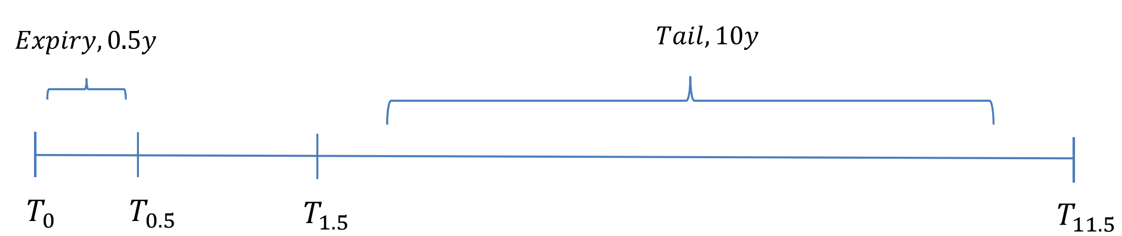
Let's consider the profile of the mid-curve option again. Replication of this profile is analogous to the replication of forward swaps previously detailed. To mimic the exposure of the 6m1y10y mid-curve option:

We buy 6m11y:



And sell 6m1y:

Leaving us with exposure to the 6m1y10y mid-curve:



Thus, one can approximately replicate a -60k/nv exposure on the 6m1y10y mid-curve by:

-Sell 66k/nv 6m11y
-Buy 6k/nv 6m1y

This vanilla replication is an approximation as it fails to incorporate the correlation exposure from the mid-curve. (This too will be addressed in the Appendix.)

In summary, we have developed an intuitive framework for the pricing of a Forward Volatility Agreement. As per our example of buying a 6m1y10y FVA, we have effectively created a long position in 18m10y (our predominant exposure), a short position in 6m11y, and a long position in 6m1y. While this is not a closed-form solution, it does capture the main risk vectors and identifies the general risk structure. The Appendix will now further detail the mathematics behind this replication process.

Appendix

For readers with backgrounds in other asset classes who are not accustomed to seeing normal volatility, we include a brief note on the topic. We measure the diffusion process in terms of absolute changes (basis point moves in the forward rate), signifying $r_t - r_{t-1}$ ~N(μ, σ^2). Thus, σ is notation for the standard deviation of absolute changes in the forward rate, in basis points per year.

Calculating the Implied Forward Volatility of the FVA

We generalize our framework below.

We define an FVA as a straddle that is struck at time T_1 and expires on T_2 into a swap that matures on T_3. The mid-curve in our replication is struck on T_0, expiring on T_1into a swap starting on T_2 and maturing on T_3.

As variance is proportional to time, it follows that:

$$(T_2 - T_1)\sigma_{FVA}^2 = (T_2 - T_0)\sigma_{T_2,T_3}^2 - (T_1 - T_0)\sigma_{MC}^2$$

The implied volatility of the mid-curve can be calculated using the DV01-weighted spread option formula. If $DV01_{T1,T2}$ is the DV01 of a swap that starts on T_1 and matures on T_2:

$$\sigma_{MC} = \sqrt{\frac{DV01_{T1,T3}^2\sigma_{T1,T3}^2 + DV01_{T1,T2}^2\sigma_{T1,T2}^2 - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}\sigma_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}{DV01^2{}_{T2,T3}}}$$

The only variable that is not readily transparent in the market is the implied correlation between the T_1, T_2 and T_1, T_3 rates. One could calculate this level using historical delivered correlation, or back out the market implied correlation from a T_1expiry option on the $T_3 - T_1, T_2 - T_1$ spread. Practically, the forward volatility of the mid-curve trades above the level implied using a correlation of one (partially because this replication approach ignores skew effects that give value to the forward volatility.)

Using the above formulas, we can calculate the implied forward volatility using only vanilla options and the DV01s of the forwards:

$$\sigma_{FVA} = \sqrt{\frac{(T_2 - T_0)\sigma_{T_2,T_3}^2 - (T_1 - T_0)\dfrac{DV01_{T1,T3}^2\sigma_{T1,T3}^2 + DV01_{T1,T2}^2\sigma_{T1,T2}^2 - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}\sigma_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}{DV01^2{}_{T2,T3}}}{T_2 - T_1}}$$

Vega Calculations:

The price of the FVA is equal to:

$$P_{FVA} = \sqrt{\frac{2}{\pi}}\sqrt{T_2 - T_1}\,\sigma_{FVA}\;\;DV01_{T2,T3}\,(1 + \frac{r_{T0,T1}}{2})^{2(T_1 - T_0)}$$

Thus, the vega is:

$$\frac{\partial P_{FVA}}{\partial \sigma_{FVA}} = \sqrt{\frac{2}{\pi}}\sqrt{T_2 - T_1}\;\;DV01_{T2,T3}\,(1 + \frac{r_{T0,T1}}{2})^{2(T_1 - T_0)}$$

Using the chain rule, we can calculate the sensitivity of the normal volatility to a change in implied normal volatility of the vanilla and the mid-curve:

$$\frac{\partial \sigma_{FVA}}{\partial \sigma_{T2,T3}} = \frac{1}{2}\sqrt{\frac{T_2 - T_1}{(T_2 - T_0)\sigma_{2,3}^2 - (T_1 - T_0)\sigma_{MC}^2}}\ \frac{2(T_2 - T_0)\sigma_{2,3}}{T_2 - T_1}$$

$$\frac{\partial \sigma_{FVA}}{\partial \sigma_{MC}} = \frac{1}{2}\sqrt{\frac{T_2 - T_1}{(T_2 - T_0)\sigma_{2,3}^2 - (T_1 - T_0)\sigma_{MC}^2}}\ \frac{2(T_1 - T_0)\sigma_{MC}}{T_2 - T_1}$$

Thus, the normal vega is:

$$\frac{\partial P_{FVA}}{\partial \sigma_{T2,T3}} = \frac{1}{2}\sqrt{\frac{T_2 - T_1}{(T_2 - T_0)\sigma_{2,3}^2 - (T_1 - T_0)\sigma_{MC}^2}}\ \frac{2(T_2 - T_0)\sigma_{2,3}}{T_2 - T_1}\sqrt{\frac{2}{\pi}} * \sqrt{T_2 - T_1} * DV01_{T2,T3} * (1 + \frac{r_{T0,T1}}{2})^{2(T_1 - T_0)}$$

$$\frac{\partial P_{FVA}}{\partial \sigma_{MC}} = \frac{1}{2}\sqrt{\frac{T_2 - T_1}{(T_2 - T_0)\sigma_{2,3}^2 - (T_1 - T_0)\sigma_{MC}^2}}\ \frac{2(T_1 - T_0)\sigma_{MC}}{T_2 - T_1}\sqrt{\frac{2}{\pi}} * \sqrt{T_2 - T_1} * DV01_{T2,T3} * (1 + \frac{r_{T0,T1}}{2})^{2(T_1 - T_0)}$$

We apply the same logic to generate vega sensitivities of the vanillas to replicate the mid-curve option:

$$\frac{\partial \sigma_{MC}}{\partial \sigma_{T1,T3}} = \frac{1}{2}\frac{1}{\sqrt{DV01_{T1,T3}^2\sigma_{T1,T3}^2 + DV01_{T1,T2}^2\sigma_{T1,T2}^2 - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}\sigma_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}}\ \frac{2DV01_{T1,T3}^2\sigma_{T1,T3} - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}{DV01_{T2,T3}}$$

$$\frac{\partial \sigma_{MC}}{\partial \sigma_{T1,T2}} = \frac{1}{2}\frac{1}{\sqrt{DV01_{T1,T3}^2\sigma_{T1,T3}^2 + DV01_{T1,T2}^2\sigma_{T1,T2}^2 - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}\sigma_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}}\ \frac{2DV01_{T1,T3}^2\sigma_{T1,T2} - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}DV01_{T1,T2}\sigma_{T1,T3}}{DV01_{T2,T3}}$$

$$\frac{\partial P_{MC}}{\partial \sigma_{T1,T3}} = \frac{1}{2}\frac{1}{\sqrt{DV01^2_{T1,T3}\sigma^2_{T1,T3} + DV01^2_{T1,T2}\sigma^2_{T1,T2} - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}\sigma_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}} \frac{2DV01^2_{T1,T3}\sigma_{T1,T3} - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}{DV01_{T2,T3}}\sqrt{\frac{2}{\pi}} * \sqrt{T_1 - T_0} * DV01_{T2,T3}$$

$$\frac{\partial P_{MC}}{\partial \sigma_{T1,T2}} = \frac{1}{2}\frac{1}{\sqrt{DV01^2_{T1,T3}\sigma^2_{T1,T3} + DV01^2_{T1,T2}\sigma^2_{T1,T2} - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}\sigma_{T1,T3}DV01_{T1,T2}\sigma_{T1,T2}}} \frac{2DV01^2_{T1,T2}\sigma_{T1,T2} - 2\rho_{(T1,T3),(T1,T2)}DV01_{T1,T3}DV01_{T1,T2}\sigma_{T1,T3}}{DV01_{T2,T3}}\sqrt{\frac{2}{\pi}} * \sqrt{T_1 - T_0} * DV01_{T2,T3}$$

An Example: 6m1y10y

We return to our initial example: A Customer would like to purchase 100k/nv exposure to a 6m1y10y FVA.

At this rate level, the DV01 of the 18m10y rate is 8.6, that's equivalent to approximately 146mm notional of the FVA.

Let's make the following volatility surface assumptions: The normal volatility for the 6m1y is 37bp, the 6m11y is 99bp, the 1y10y is 100bp, and the 18m10y is 99bp. Additionally, the DV01 of the 6m1y is 1.0 and the DV01 of 6m11y is 9.6. For simplicity, we use a 6-month discount factor of one. Finally, we assume a correlation of one for the mid-curve.

Using these assumptions, the normal volatility of the 6m1y10y mid-curve is 107bp; where $\frac{\partial \sigma_{FVA}}{\partial \sigma_{18m10y}}$ is 1.6 and $\frac{\partial \sigma_{FVA}}{\partial \sigma_{MC}}$ is 0.6. So for a 100k/nv exposure FVA, we need to buy 160k 18m10y and sell 60k of the 6m1y10y mid-curve. Since $\frac{\partial P_{18m10y}}{\partial \sigma_{18m10y}}$ is 8.4bp, we need to buy 186mm 18m10y. We calculate that $\frac{\partial P_{MC}}{\partial \sigma_{MC}}$ is 4.9bp, so we need to sell 116mm of the mid-curve.

This leads to the ultimate value added proposition – when the Spot Implied Volatility of a 1y10y is trading at 100nv, a client can purchase it six months forward at 95nv, a 5% discount to the spot level.

On a practical note, we should highlight that most clients trade notionally neutral in the vanilla and the mid-curve fixed-strike options when attempting a replication. While this is fine for a terminal replication, it does not match the actual vega exposure of the FVA at inception; as such, there can be considerable mark-to-market variation.

If you have fully absorbed this appendix and sample application, we look forward to helping you duplicate our modeling process. If instead you have just paged to the end looking to see if we have any new colors to offer, we will just close by reminding you that the FVA is a truly special product in that it allows an investor to gain exposure to *Volatility as an Asset Class* (executed on your existing ISDA) without the need to engage in a costly and complicated "replication process".

In a nutshell, the FVA is just plain old-fashioned Financial Engineering in the classic manner of traditional Wall Street.

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